### Electric Potential, Potential Energy, and Force

**Learning Goal:** To review relationships among electric potential, electric potential energy, and force on a test charge.

This problem is a review of the relationship between an electric field \( \vec{E} \), its associated electric potential \( V \), the electric potential energy \( U \) and the direction of force on a test charge.

#### Part A

Electric field lines always begin at _______ charges (or at infinity) and end at _______ charges (or at infinity). One could also say that the lines we use to represent an electric field indicate the direction in which a _______ test charge would initially move when released from rest. Which of the following fills in the three missing words correctly?

**ANSWER:**
- (positive; negative; negative)
- (positive; negative; positive)
- (negative; positive; positive)
- (negative; positive; negative)

Note that the electric field vector \( \vec{E} \) is everywhere tangent to the electric field lines. Like electric field lines, the electric field vector generally points away from positive charges and toward negative charges.

#### Part B

Would a positive test charge released from rest move toward a region of higher or lower electric potential (compared to the electric potential at the point where it is released)?

**Hint B.1** Potential, field, and force

**ANSWER:**
- higher electric potential
- lower electric potential

**Correct**

#### Part C

Now imagine that the sign of our test particle is changed from positive to negative, but the electric potential remains the same. Which of the following statements is correct?

**Hint C.1** Direction of field and force

**Hint not displayed**

**Hint C.2** Direction of force and potential energy gradient

**Hint not displayed**

**ANSWER:**
- The direction of the force will change and it will point to regions of higher potential energy.
- The direction of the force will not change and it will point to regions of higher potential energy.
- The direction of the force will not change and it will point to regions of lower potential energy.
- The direction of the force will change and it will point to regions of lower potential energy.

**Correct**

### Problem 30.37

The electric field in a region of space is \( \vec{E}_x = -1900 \, \text{V/m} \), where \( x \) is in meters.

#### Part A

What is the potential difference between \( x_1 = -10 \, \text{cm} \) and \( x_2 = 90 \, \text{cm} \)?

**Express your answer using two significant figures.**

**ANSWER:**
\[ \Delta V = 760 \, \text{V} \]

**Correct**

### Problem 30.47

The electric potential in a region of space is \( V = (210 \, x^2 - 100 \, y^2) \, \text{V} \), where \( x \) and \( y \) are in meters.

#### Part A

What is the strength of the electric field at \( (x, y) = (3.0 \, \text{m}, 3.0 \, \text{m}) \)?

**Express your answer using two significant figures.**

**ANSWER:**
\[ E = 1400 \, \text{V/m} \]

**Correct**
Part B
What is the direction of the electric field at \((x, y) = (3.0 \text{ m}, 3.0 \text{ m})\)? Give the direction as an angle (in degrees) counterclockwise from the positive \(x\)-axis.

\[
\theta = 155^\circ \quad \text{Correct counterclockwise from the positive } x\text{-axis}
\]

Capacitance: A Review

Learning Goal: To review the meaning of capacitance and ways of changing the capacitance of a parallel-plate capacitor.

Capacitance is one of the central concepts in electrostatics. Understanding its meaning and the difference between its definition and the ways of calculating capacitance can be challenging at first. This tutorial is meant to help you become more comfortable with capacitance. Recall the fundamental formula for capacitance:

\[
C = \frac{Q}{V}
\]

where \(C\) is the capacitance in farads, \(Q\) is the charge stored on the plates in coulombs, and \(V\) is the potential difference (or voltage) between the plates. In the following problems it may help to keep in mind that the voltage is related to the strength of the electric field \(E\) and the distance between the plates, \(d\), by

\[
V = Ed
\]

Part A
What property of objects is best measured by their capacitance?

\[
\text{Correct: the ability to store charge}
\]

Capacitance is a measure of the ability of a system of two conductors to store electric charge and energy. It is defined as \(C = \frac{Q}{V}\). This ratio remains constant as long as the system retains its geometry and the amount of dielectric does not change. Capacitors are special devices designed to combine a large capacitance with a small size. However, any pair of conductors separated by a dielectric (or vacuum) has some capacitance. Even an isolated electrode has a small capacitance. That is, if a charge \(Q\) is placed on it, its potential \(V\) with respect to ground would change, and the ratio \(Q/V\) is its capacitance \(C\).

Part B
Consider an air-filled charged capacitor. How can its capacitance be increased?

\[
\text{Correct: Increase the charge on the capacitor.}
\]

Part C
Consider a charged parallel-plate capacitor. How can its capacitance be halved?

\[
\text{Correct: Halve the plate separation.}
\]

Part D
Consider a charged parallel-plate capacitor. Which combination of changes would quadruple its capacitance?

\[
\text{Correct: Double the charge and double the plate area.}
\]

Problem 30.19
A switch that connects a battery to a 10 \mu F capacitor is closed. Several seconds later you find that the capacitor plates are charged to \pm 30 \mu C.

**Part A**
What is the emf of the battery?

**ANSWER:**

\[
\begin{array}{c}
3.00 \\
Correct
\end{array}
\]

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**Equivalent Capacitance**

Consider the combination of capacitors shown in the diagram, where \( C_1 = 3.00 \ \mu F \), \( C_2 = 11.0 \ \mu F \), \( C_3 = 3.00 \ \mu F \), and \( C_4 = 5.00 \ \mu F \).

**Part A**
Find the equivalent capacitance \( C_A \) of the network of capacitors.

**Hint A.1** How to reduce the network of capacitors

**Hint not displayed**

**Hint A.2** Find the capacitance equivalent to \( C_2 \) \( C_3 \) and \( C_4 \)

**Hint not displayed**

**Hint A.3** Two capacitors in series

**Hint not displayed**

Express your answer in microfarads.

**ANSWER:**

\[
C_A = 2.59 \ \mu F
\]

**Correct**

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**Part B**
Two capacitors of capacitance \( C_5 = 6.00 \ \mu F \) and \( C_6 = 3.00 \ \mu F \) are added to the network, as shown in the diagram. Find the equivalent capacitance \( C_B \) of the new network of capacitors.

**Hint B.1** How to reduce the extended network of capacitors

**Hint not displayed**

**Hint B.2** Find the equivalent capacitance of \( C_2 \) \( C_3 \) \( C_5 \) and \( C_6 \)

**Hint not displayed**

**Hint B.3** Two capacitors in series

**Hint not displayed**

Express your answer in microfarads.

**ANSWER:**

\[
C_B = 2.54 \ \mu F
\]

**Correct**

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**Problem 30.25**

You need a capacitance of 50.0 \ \mu F, but you don't happen to have a 50.0 \ \mu F capacitor. You do have a 75.0 \ \mu F capacitor.

**Part A**
What additional capacitor do you need to produce a total capacitance of 50.0 \ \mu F?

**ANSWER:**

\[
150 \ \mu F
\]
Part B
Should you join the two capacitors in parallel or in series?

**ANSWER:**
- in parallel
- in series

Correct

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**A Simple Network of Capacitors**

In the figure are shown three capacitors with capacitances $C_1 = 6.00 \, \mu F$, $C_2 = 3.00 \, \mu F$, $C_3 = 5.00 \, \mu F$. The capacitor network is connected to an applied potential $V_{ap}$. After the charges on the capacitors have reached their final values, the charge $Q_2$ on the second capacitor is $40.0 \, \mu C$.

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**Part A**

What is the charge $Q_1$ on capacitor $C_1$?

**Hint A.1** How to approach the problem

**Hint not displayed**

**Hint A.2** Series or parallel?

**Hint not displayed**

**Hint A.3** Calculate the potential difference across the second capacitor

**Hint not displayed**

**Hint A.4** Calculate the potential difference across the first capacitor, $V_1$

**Hint not displayed**

**ANSWER:**

$Q_1 = 80.0 \, \mu C$

Correct

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**Part B**

What is the charge on capacitor $C_2$?

**Hint B.1** How to approach the problem

**Hint not displayed**

**Hint B.2** Series or parallel?

**Hint not displayed**

**Hint B.3** Capacitors in series

**Hint not displayed**

**Hint B.4** Calculate the total charge in the initial section

**Hint not displayed**

Express your answer in microcoulombs to three significant figures.

**ANSWER:**

$Q_2 = 120 \, \mu C$

Correct

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**Part C**

What is the applied voltage, $V_{ap}$?

**Hint C.1** How to approach the problem

**Hint not displayed**

**Hint C.2** Calculate the equivalent capacitance

**Hint not displayed**

**Hint C.3** Find the total charge

**Hint not displayed**

Express your answer in volts to three significant figures.
Problem 30.61

What are the charge on and the potential difference across each capacitor in the figure?

Part A

\[ Q_1 \]

**ANSWER:**

4.00 μC

Correct

Part B

\[ V_1 \]

**ANSWER:**

1.00 V

Correct

Part C

\[ Q_2 \]

**ANSWER:**

12.0 μC

Correct

Part D

\[ V_2 \]

**ANSWER:**

1.00 V

Correct

Part E

\[ Q_3 \]

**ANSWER:**

16.0 μC

Correct

Part F

\[ V_3 \]

**ANSWER:**

8.00 V

Correct

Problem 30.73

The flash unit in a camera uses a 3.0 V battery to charge a capacitor. The capacitor is then discharged through a flashlamp. The discharge takes 10 ms and the average power dissipated in the flashlamp is 10 W.

Part A

What is the capacitance of the capacitor?

**ANSWER:**

22.2 μF

Correct

Problem 30.33

Two 4.6 cm-diameter metal disks are separated by a 0.23 mm-thick piece of paper. (Dielectric constant for paper is \( \kappa_{\text{paper}} = 3.7 \), its dielectric strength is 16 \( \times 10^6 \) V/m.)

Part A

What is the capacitance?

**ANSWER:**

\[ C = 0.24 \text{ µF} \]

Correct

Part B
The Capacitor as an Energy-Storing Device

Learning Goal:
To understand that the charge stored by capacitors represents energy; to be able to calculate the stored energy and its changes under different circumstances.

An air-filled parallel-plate capacitor has plate area \( A \) and plate separation \( d \). The capacitor is connected to a battery that creates a constant voltage \( V \).

**Part A**

Find the energy \( U_0 \) stored in the capacitor.

**Hint A.1** Formula for the energy of a capacitor

Express your answer in terms of \( A \), \( d \), \( V \) and \( \epsilon_0 \). Remember to enter \( \epsilon_0 \) as \( \epsilon_0 \).

**ANSWER:**

\[
\frac{1}{2} \epsilon_0 \frac{A}{d} V^2 \]

**Correct**

**Part B**

The capacitor is now disconnected from the battery, and the plates of the capacitor are then slowly pulled apart until the separation reaches \( 3d \). Find the new energy \( U_1 \) of the capacitor after this process.

**Hint B.1** What quantity remains constant?

What characteristic of the capacitor does not change in this process?

**ANSWER:**

\( \text{charge} \)

\( \text{voltage between the plates} \)

\( \text{capacitance} \)

\( \text{energy} \)

**Correct**

In particular, look carefully at the quantities that do change.

**Hint B.2** Find the charge on the capacitor

What is the charge \( Q \) that resides on the plates of the capacitor?

Express your answer in terms of some or all of the variables \( V', A, \) and \( d \). Remember to enter \( \epsilon_0 \) as \( \epsilon_0 \).

**ANSWER:**

\[
Q = \frac{\epsilon_0 A V}{3d} \]

**Correct**

**Hint B.3** How does the capacitance change?

How does the capacitance change in this process?

**ANSWER:**

\( \text{remains constant} \)

\( \text{increases by a factor of 3} \)

\( \text{decreases by a factor of 3} \)

\( \text{increases by a factor of 9} \)

\( \text{decreases by a factor of 9} \)

**Correct**

**Hint B.4** What is the formula for the energy?

Which of the following formulas is most useful in finding \( U_1 \) for this situation?

**ANSWER:**

\[ \frac{CV^2}{2} \]

\[ \frac{QV}{2} \]

\[ \frac{Q^2}{2C} \]

**Correct**

Express your answer in terms of \( A \), \( d \), \( V' \) and \( \epsilon_0 \).

**ANSWER:**

\[
U_1 = \frac{3}{2} \frac{\epsilon_0 A V^2}{2d} \]
The increase in energy of the capacitor comes from the external work that must be done to pull the plates apart. Keep in mind that the plates have opposite charges and attract each other; some work must be done by an external agent to pull them apart.

**Part C**

The capacitor is now reconnected to the battery, and the plate separation is restored to $d$. A dielectric plate is slowly moved into the capacitor until the entire space between the plates is filled. Find the energy $U_d$ of the dielectric-filled capacitor. The capacitor remains connected to the battery. The dielectric constant is $K$.

Express your answer in terms of $A$, $d$, $V$, $K$, and $\varepsilon_0$.

**ANSWER:**

$$U_d = \frac{(K \varepsilon_0 V^2)}{2d}$$