Chapter 21 Homework
Due: 8:00am on Tuesday, January 26, 2010

Note: To understand how points are awarded, read your instructor's Grading Policy.

Conceptual Question 21.1

The figure shows a standing wave oscillating on a string at frequency \( f_0 \).

Part A
What mode (\( n \) value) is this?

ANSWER: \( n = 4 \) Correct

Part B
How many antinodes will there be if the frequency is increased to \( 8f_0 \)?

ANSWER: \( n = 32 \) Correct 32 antinodes

Harmonics of a Piano Wire

A piano tuner stretches a steel piano wire with a tension of 765 N. The steel wire has a length of 0.600 m and a mass of 4.50 g.

Part A
What is the frequency \( f_1 \) of the string's fundamental mode of vibration?

Hint A.1 How to approach the problem

Hint not displayed

Hint A.2 Find the mass per unit length

Hint not displayed

Hint A.3 Equation for the fundamental frequency of a string under tension

Hint not displayed

Express your answer numerically in hertz using three significant figures.

ANSWER: \( f_1 = 266 \) Correct

Part B
What is the number of the highest harmonic that could be heard by a person who is capable of hearing frequencies up to 6521 Hz?

Hint B.1 Harmonics of a string

The harmonics of a string are given by \( f_n = nf_1 \), where \( f_n \) is the \( n \)th harmonic of a string with fundamental frequency \( f_1 \). Be careful if you get a noninteger answer for \( n \), as harmonics can only be integer multiples of the fundamental frequency.

ANSWER: \( n = 24 \) Correct

When solving this problem, you may have found a noninteger value for \( n \), but harmonics can only be integer multiples of the fundamental frequency.

Standing Waves on a Guitar String

Learning Goal: To understand standing waves, including calculation of \( \lambda \) and \( f \) and to learn the physical meaning behind some musical terms.

The columns in the figure show the instantaneous shape of a vibrating guitar string drawn every 1 ms. The guitar string is 60 cm long.

The left column shows the guitar string shape as a sinusoidal traveling wave passes through it. Notice that the shape is sinusoidal at all times and specific features, such as the crest indicated with the arrow, travel along the string to the right at a constant speed.

The right column shows snapshots of the sinusoidal standing wave formed when this sinusoidal traveling wave passes through an identically shaped wave moving in the opposite direction on the same guitar string. The string is momentarily flat when the underlying traveling waves are exactly out of phase. The shape is sinusoidal with twice the original amplitude when the underlying waves are momentarily in phase. This pattern is called a standing wave because no wave features travel down the length of the string.
Standing waves on a guitar string form when waves traveling down the string reflect off a point where the string is tied down or pressed against the fingerboard. The entire series of distortions may be superimposed on a single figure, like this, indicating different moments in time using traces of different colors or line styles.

Part A
What is the wavelength $\lambda$ of the standing wave shown on the guitar string?

Hint A.1 Identify the wavelength of a sinusoidal shape
The wavelength of a sinusoidal shape is the distance from a given feature to the next instance of that same feature. Wavelengths are usually measured from one peak to the next peak. What is the wavelength $\lambda$ of this sinusoidal pattern?

Express your answer in centimeters.

\[ \lambda = 20 \text{ cm} \]

Correct

Now answer the original question by considering the guitar string at a moment when it looks sinusoidal, not flat.

Express your answer in centimeters.

\[ \lambda = 40 \text{ cm} \]

Correct

Nodes are locations in the standing wave pattern where the string doesn't move at all, and hence the traces on the figure intersect. In between nodes are antinodes, where the string moves up and down the most.

This standing wave pattern has three antinodes, at $x = 10$ cm, $30$ cm, and $50$ cm. The pattern also has four nodes, at $x = 0$ cm, $20$ cm, $40$ cm, and $60$ cm. Notice that the spacing between adjacent antinodes is only half of one wavelength, not one full wavelength. The same is true for the spacing between adjacent nodes.

This figure shows the first three standing wave patterns that fit on any string with length $L$ tied down at both ends. A pattern's number $n$ is the number of antinodes it contains. The wavelength of the $n$th pattern is denoted $\lambda_n$. The $n$th pattern has $\frac{n}{2}$ half-wavelengths along the length of the string, so

\[ \frac{\lambda_n}{2} = L \]

Thus the wavelength of the $n$th pattern is

\[ \lambda_n = \frac{2L}{n} \]

Part B
What is the wavelength of the longest wavelength standing wave pattern that can fit on this guitar string?

Hint B.1 How to approach the problem

Hint B.2 Determine $n$ for the longest wavelength pattern
What is the pattern number $n$ for the longest wavelength standing wave pattern?

\[ \begin{align*}
&n = 1 \\
&n = 2 \\
&n = 3 \\
&\text{some other integer}
\end{align*} \]

Correct

Express your answer in centimeters.

\[ \lambda_n = 120 \text{ cm} \]
Part A

How does the overtone number relate to the standing wave pattern number, previously denoted with the variable \( n \)?

Waves of all wavelengths travel at the same speed \( v \) on a given string. Traveling wave velocity and wavelength are related by

\[
v = \lambda f,
\]

where \( v \) is the wave speed (in meters per second), \( \lambda \) is the wavelength (in meters), and \( f \) is the frequency (in inverse seconds, also known as hertz (Hz)).

Since only certain wavelengths fit properly to form standing waves on a specific string, only certain frequencies will be represented in that string's standing wave series. The frequency of the \( n \)th pattern is

\[
f_n = \frac{v}{2L} = \frac{v}{2 \lambda_n} = \frac{n v}{2L}.
\]

Note that the frequency of the fundamental is \( f_1 = v/(2L) \), so \( f_n \) can also be thought of as an integer multiple of \( f_1 \): \( f_n = nf_1 \).

Part C

The frequency of the fundamental of the guitar string is 320 Hz. At what speed \( v \) do waves move along that string?

Express your answer in meters per second.

**ANSWER:**

\[ v = 384 \text{ m/s} \] (Correct)

Notice that these transverse waves travel slightly faster than the speed of sound waves in air, which is about 340 m/s.

We are now in a position to understand certain musical terms from a physics perspective.

The standing wave frequencies for this string are \( f_1 = 320 \text{ Hz} \), \( f_2 = f_1 = 640 \text{ Hz} \), \( f_3 = 3f_1 = 960 \text{ Hz} \), etc. This set of frequencies is called a harmonic series and it contains common musical intervals such as the octave (in which the ratio of frequencies of the two notes is 2:1) and the perfect fifth (in which the ratio of frequencies of the two notes is 3:2). Here \( f_1 \) is one octave above \( f_2 \), \( f_2 \) is a perfect fifth above \( f_3 \), and so on. Standing wave patterns with frequencies higher than the fundamental frequency are called overtones. The \( n = 2 \) pattern is called the first overtone, the \( n = 3 \) pattern is called the second overtone, and so on.

Part D

How does the overtone number relate to the standing wave pattern number, previously denoted with the variable \( n \)?

**ANSWER:**

- overtone number = pattern number
- overtone number = pattern number + 1
- overtone number = pattern number - 1
- There is no strict relationship between overtone number and pattern number.

The overtone number and the pattern number are easy to confuse but they differ by one. When referring to a standing wave pattern using a number, be explicit about which numbering scheme you are using.

When you pluck a guitar string, you actually excite many of its possible standing waves simultaneously. Typically, the fundamental is the loudest, so that is the pitch you hear. However, the unique mix of the fundamental plus overtones is what makes a guitar sound different from a violin or a flute, even if they are playing the same note (i.e., producing the same fundamental). This characteristic of a sound is called its timbre (rhymes with amber).

A sound containing just a single frequency is called a pure tone. A complex tone, in contrast, contains multiple frequencies such as a fundamental plus some of its overtones. Interestingly enough, it is possible to fool someone into identifying a frequency that is not present by playing just its overtones. For example, consider a sound containing pure tones at 450 Hz, 600 Hz, and 750 Hz. Here 600 Hz and 750 Hz are not integer multiples of 450 Hz, so 450 Hz would not be considered the fundamental with the other two as overtones. However, because all three frequencies are consecutive overtones of 150 Hz a listener might claim to hear 150 Hz over an octave below any of the frequencies present. This 150 Hz is called a virtual pitch or a missing fundamental.

Part E

A certain sound contains the following frequencies: 400 Hz, 1600 Hz, and 2400 Hz. Select the best description of this sound.

**ANSWER:**

- This is a pure tone.
- This is a complex tone with a fundamental of 400 Hz plus some of its overtones.
- This is a complex tone with a virtual pitch of 800 Hz.
- These frequencies are unrelated, so they are probably pure tones from three different sound sources.

These concepts of fundamentals and overtones can be applied to other types of musical instruments besides string instruments. Hollow-tube instruments, such as brass instruments and reed instruments, have standing wave patterns in the air within them. Percussion instruments, such as bells and cymbals, often exhibit standing wave vibrations in the solid material of their bodies. Even the human voice can be analyzed this way, with the fundamental setting the pitch of the voice and the presence or absence of overtones setting the unique vowel or consonant being sounded.

**Why the Highest Piano Notes Have Short Strings**

The steel used for piano wire has a breaking (tensile) strength \( p_y \) of about 4 x 10^6 N/m^2 and a density \( \rho \) of 7800 kg/m^3.

**Part A**
What is the speed $v$ of a wave traveling down such a wire if the wire is stretched to its breaking point?

**Hint A.1** Find the traveling wave speed in a stretched string

**Hint not displayed**

**Hint A.2** Express $v$ in terms of tensile strength and density

**Hint not displayed**

Express the speed of the wave numerically, in meters per second, to the nearest integer.

**ANSWER:** $v = 620$ m/s

This is much less than the speed of sound in steel (5941 m/s) because, unlike steel, in piano wire the tensile strength is much less than the Young's modulus.

**Part B**

Imagine that the wire described in the problem introduction is used for the highest C on a piano (C8 = 4400 Hz). If the wire is in tune when stretched to its breaking point, what must the vibrating length of the wire be?

**Hint B.1** A length constraint given $v$ and $f$

In Part A of this problem, you found the speed of traveling waves in the wire ($v$). You also know the frequency of oscillation ($f$). What is the wavelength ($\lambda$) of the wave in the C8 piano string, in terms of these quantities?

**Express your answer in terms of $v$ and $f$.**

**ANSWER:**

**Hint B.2** Relationship between wavelength and string length

Consider the boundary conditions for a stretched piano wire: Both ends are fixed. If such a wire is oscillating at its fundamental frequency (its first normal mode), the wavelengths will not be equal to the wire length. What is the wavelength of the first normal mode of a string of length $L$ that is fixed at both ends?

**Express the wavelength in terms of $L$.**

**ANSWER:**

Here is a figure that might help you remember this result.

Express the length numerically, in centimeters, using three significant figures.

**ANSWER:** $L = 7.75$ cm

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**Problem 21.29**

Two strings are adjusted to vibrate at exactly 200 Hz. Then the tension in one string is increased slightly. Afterward, three beats per second are heard when the strings vibrate at the same time.

**Part A**

What is the new frequency of the string that was tightened?

**ANSWER:** 203 Hz

**Problem 21.38**

A violinist places her finger so that the vibrating section of a 1.30 m string has a length of 30.0 cm, then she draws her bow across it. A listener nearby in a 20°C room hears a note with a wavelength of 50.0 cm.

**Part A**

What is the tension in the string?

**ANSWER:** 220 N

**Problem 21.77**

Two loudspeakers emit 400 Hz notes. One speaker sits on the ground. The other speaker is in the back of a pickup truck. You hear 6.00 beats per second as the truck drives away from you.
Introduction to Wind Instruments

The physics of wind instruments is based on the concept of standing waves. When the player blows into the mouthpiece, the column of air inside the instrument vibrates, and standing waves are produced. Although the acoustics of wind instruments is complicated, a simple description in terms of open and closed tubes can help in understanding the physical phenomena related to these instruments. For example, a flute can be described as an open-open pipe because a flutist covers the mouthpiece of the flute only partially. Meanwhile, a clarinet can be described as an open-closed pipe because the mouthpiece of the clarinet is almost completely closed by the reed.

Throughout the problem, take the speed of sound in air to be 343 m/s.

Part A
Consider a pipe of length 80.0 cm open at both ends. What is the lowest frequency of the sound wave produced when you blow into the pipe?

Express your answer in hertz.

ANSWER: \( f = 214 \) Hz

Correct

If your pipe were a flute, this frequency would be the lowest note that can be produced on that flute. This frequency is also known as the fundamental frequency or first harmonic.

Part B
A hole is now drilled through the side of the pipe and air is blown again into the pipe through the same opening. The fundamental frequency of the sound wave generated in the pipe is now

Lower than before.

Higher than before.

Correct

By opening successive holes closer and closer to the opening used to blow air into the pipe, the pipe can be made to produce sound at higher and higher frequencies. This is what flutists do when they open the tone holes on the flute.

Part C
If you take the original pipe in Part A and drill a hole at a position half the length of the pipe, what is the fundamental frequency of the sound that can be produced in the pipe?

Express your answer in hertz.

ANSWER: \( f = 429 \) Hz

Correct

Part D
What frequencies, in terms of the fundamental frequency of the original pipe in Part A, can you create when blowing air into the pipe that has a hole halfway down its length?

Recall from the discussion in Part B that the standing wave produced in the pipe must have an antinode near the hole. Thus only the harmonics that have an antinode halfway down the pipe will still be present.

ANSWER: Only the odd multiples of the fundamental frequency

Correct

Part E
What length of open-closed pipe would you need to achieve the same fundamental frequency as the open-open pipe discussed in Part A?

Express your answer in terms of the length of the open-open pipe.

ANSWER: Half the length of the open-open pipe

Correct
Part F
What is the frequency of the first possible harmonic after the fundamental frequency in the open-closed pipe described in Part E?

Hint F.1 How to approach the problem
Recall that possible frequencies of standing waves that can be generated in an open-closed pipe include only odd harmonics. Then the first possible harmonic after the fundamental frequency is the third harmonic.

Express your answer in hertz.

\[ f'' = 643 \text{ Hz} \]

FM Radio Interference

You are listening to the FM radio in your car. As you come to a stop at a traffic light, you notice that the radio signal is fuzzy. By pulling up a short distance, you can make the reception clear again. In this problem, we work through a simple model of what is happening.

Our model is that the radio waves are taking two paths to your radio antenna:
- the direct route from the transmitter
- an indirect route via reflection off a building

Because the two paths have different lengths, they can constructively or destructively interfere. Assume that the transmitter is very far away, and that the building is at a 45-degree angle from the path to the transmitter.

Point A in the figure is where you originally stopped, and point B is where the station is completely clear again. Finally, assume that the signal is at its worst at point A, and at its clearest at point B.

Part A
What is the distance between points A and B?

Hint A.1 What is the path-length difference at point A?
Since we know that the waves traveling along the two paths interfere destructively at point A, we know something about the difference in the lengths of those two paths. What is the difference between the two path lengths, in integer multiples of the wavelength?

\[ \text{ANSWER:} \quad \frac{N}{2} \]

Correct

To have destructive interference, the two waves must be shifted relative to one another by half of a wavelength. Integer numbers of wavelengths in the path difference do not change the relative positions of the waves, so the value of \( N \) is irrelevant.

Hint A.2 What is the path-length difference at point B?
Since we know that the waves traveling along the two paths interfere constructively at point B, we know something about the difference in the lengths of those two paths. What is the difference between the two path lengths, in integer multiples of the wavelength?

\[ \text{ANSWER:} \quad \frac{N}{2} \]

Correct

For constructive interference, the waves must align crest to crest, which corresponds to a path difference of zero. Since integer numbers of wavelengths in the path difference do not change the relative positions of the waves, any integer number of wavelengths in path difference also gives constructive interference.

Hint A.3 What is the path length of reflected waves?
Consider the lengths of the paths taken by the two reflected waves. Note that the path from the transmitter to the building is larger for one wave, while the path from the building to the antenna is larger for the other. What is the difference in length between the path of the reflected wave from the transmitter to A, and the path of the reflected wave from the transmitter to B, in integer multiples of the wavelength?

\[ \text{ANSWER:} \quad \frac{N}{2} \]

Correct
Express your answer in wavelengths, as a fraction.

\[ \frac{N + \frac{1}{2}}{N} \]

Correct

ANSWER: \[ d = 0.500 \text{ Correct wavelengths} \]

Part B

Your FM station has a frequency of 100 megahertz. The speed of light is about \( 3.00 \times 10^8 \) meters per second. What is the distance \( d \) between points A and B?

Express your answer in meters, to two significant figures.

\[ d = 1.5 \text{ Correct} \]

Interference of Sound Waves

Two loudspeakers, A and B, are driven by the same amplifier and emit sinusoidal waves in phase. The frequency of the waves emitted by each speaker is 172 Hz. You are 8.00 m from speaker A. Take the speed of sound in air to be 344 m/s.

Part A

What is the closest you can be to speaker B and be at a point of destructive interference?

Hint A.1 How to approach the problem

Hint not displayed

Hint A.2 Find the wavelength of the sound wave

Find the wavelength \( \lambda \) of the sound wave emitted by the loudspeakers. Take the speed of sound in air to be 344 m/s.

\[ \lambda = 2.00 \text{ Correct} \]

Hint A.2.1 Relationship between the wavelength and frequency of a periodic wave

Hint not displayed

Express your answer in meters.

\[ \lambda = 2.00 \text{ Correct} \]

Hint A.3 Find the condition for destructive interference

In general, if \( d_A \) and \( d_B \) are the paths traveled by two waves of equal frequency that are originally emitted in phase, the condition for destructive interference is

\[ d_A - d_B = n\lambda \]

where \( \lambda \) is the wavelength of the sound waves and \( n \) is any nonzero odd integer.

Given this condition for destructive interference and the situation described in the introduction of this problem, then, what is the value of \( n \) that corresponds to the shortest distance \( d_B / \)

\[ n = 7 \text{ Answer Requested} \]

Express your answer in meters.

\[ \lambda = 1.00 \text{ Correct} \]

Thin Film (Oil Slick)

A scientist notices that an oil slick floating on water when viewed from above has many different rainbow colors reflecting off the surface. She aims a spectrometer at a particular spot and measures the wavelength to be 750 nm (in air). The index of refraction of water is 1.33.

Part A

The index of refraction of the oil is 1.20. What is the minimum thickness \( t \) of the oil slick at that spot?

Hint A.1 Thin-film interference

Hint not displayed

Hint A.2 Path-length phase difference

Hint not displayed

Hint A.3 Phase shift due to reflections

Hint not displayed

Express your answer in nanometers to three significant figures.

\[ t = 313 \text{ Correct} \]
Part B
Suppose the oil had an index of refraction of 1.50. What would the minimum thickness be now?

Hint B.1 Phase shift due to reflections
Keep in mind that when light reflects off a surface with a higher index of refraction, it gains an extra shift of half of a wavelength. What used to be a maximum is now a minimum! Be careful, though; if two beams reflect, they will both get a half-wavelength shift, canceling out that effect. Also, reflection off a surface with a lower index of refraction yields no phase shift.

Express your answer in nanometers to three significant figures.

ANSWER: \( t = 125 \text{ nm} \) Correct

Part C
Now assume that the oil had a thickness of 200 nm and an index of refraction of 1.5. A diver swimming underneath the oil slick is looking at the same spot as the scientist with the spectrometer. What is the longest wavelength in water that is transmitted most easily to the diver?

Hint C.1 How to approach the problem
For transmission of light, the same rules hold as before, only now one beam travels straight through the oil slick and into the water, while the other beam reflects twice (once off the oil-water interface and once again off the oil-air interface) before being finally transmitted to the water.

Hint C.2 Determine the wavelength of light in air
Find the wavelength of the required light in air.

Express your answer numerically in nanometers.

ANSWER: \( \lambda_{\text{air}} = 600 \text{ nm} \) Correct

Hint C.3 Relationship between wavelength and index of refraction
There is a simple relationship between the wavelength of light in one medium (with one index of refraction \( n_1 \)) and the wavelength \( \lambda_2 \) in another medium (with a different index of refraction \( n_2 \)):

\[ n_1 \lambda_1 = n_2 \lambda_2 \]

Express your answer in nanometers to three significant figures.

ANSWER: \( \lambda_{\text{transmit}} = 451 \text{ nm} \) Correct

This problem can also be approached by finding the wavelength with the minimum reflection. Conservation of energy ensures that maximum transmission and minimum reflection occur at the same time (i.e., if the energy did not reflect, then it must have been transmitted to conserve energy), so finding the wavelength of minimum reflection must give the same answer as finding the wavelength of maximum transmission. In some cases, working the problem one way may be substantially easier, so you should keep both approaches in mind.

Problem 21.41
Astronauts visiting Planet X have a 2.60 m long string whose mass is 5.10 kg. They tie the string to a support, stretch it horizontally over a pulley 1.70 m away, and hang a 1.60 kg mass on the free end. Then the astronauts begin to excite standing waves on the string. Their data show that standing waves exist at frequencies of \( f_2 \) and \( f_3 \), but at no frequencies in between.

Part A
What is the value of \( g \), the acceleration due to gravity, on Planet X?

ANSWER: \( g = 3.63 \text{ m/s}^2 \) Correct

Problem 21.75
A flutist assembles her flute in a room where the speed of sound is 340 m/s. When she plays the note A, it is in perfect tune with a 440 Hz tuning fork. After a few minutes, the air inside her flute has warmed to where the speed of sound is 343 m/s.

Part A
How many beats per second will she hear if she now plays the note A as the tuning fork is sounded?

ANSWER: \( f = 3.93 \) beats Correct

Part B
How far does she need to extend the “tuning joint” of her flute to be in tune with the tuning fork?

ANSWER: \( x = 3.41 \text{ mm} \) Correct

Score Summary:
Your score on this assignment is 111.1%.
You received 78.87 out of a possible total of 80 points, plus 10 points of extra credit.