Chapter 2 Homework

Due: 9:00am on Tuesday, September 8, 2009

Note: To understand how points are awarded, read your instructor's Grading Policy.

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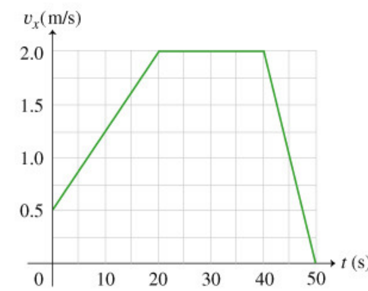
What Velocity vs. Time Graphs Can Tell You

A common graphical representation of motion along a straight line is the *v vs. t graph*, that is, the graph of (instantaneous) velocity as a function of time. In this graph, time t is plotted on the horizontal axis and velocity v on the vertical axis. Note that by definition, velocity and acceleration are vector quantities. In straight-line motion, however, these vectors have only a single nonzero component in the direction of motion. Thus, in this problem, we will call v the velocity and a the acceleration, even though they are really the components of the velocity and acceleration vectors in the direction of motion, respectively.

Here is a plot of velocity versus time for a particle that travels along a straight line with a varying velocity. Refer to this plot to answer the following questions.



Part A

What is the initial velocity of the particle, v_0?

Hint A.1 **Initial velocity**

Hint not displayed

Hint A.2 **How to read a v vs. t graph**

Hint not displayed

Express your answer in meters per second.

ANSWER: v_0 = **0.5** ***Correct*** m/s

Part B

What is the total distance Δx traveled by the particle?

Hint B.1 **How to approach the problem**

Recall that the area of the region that extends over a time interval Δt under the v vs. t curve is always equal to the distance traveled in Δt. Thus, to calculate the total distance, you need to find the area of the entire region under the v vs. t curve. In the case at hand, the entire region under the v vs. t curve is not an elementary geometrical figure, but rather a combination of triangles and rectangles.

Hint B.2 **Find the distance traveled in the first 20.0 seconds**

What is the distance Δx_1 traveled in the first 20 seconds of motion, between $t = 0.0\ \mathrm{s}$ and $t = 20.0\ \mathrm{s}$?

Hint B.2.1 **Area of the region under the v vs. t curve**

Hint not displayed

Express your answer in meters.

ANSWER: Δx_1 = **25** ***Correct*** m

Hint B.3 **Find the distance traveled in the second 20.0 seconds**

Hint not displayed

Hint B.4 **Find the distance traveled in the last 10.0 seconds**

Hint not displayed

Express your answer in meters.

ANSWER: Δx = **75** ***Correct*** m

Part C

What is the average acceleration a_{av} of the particle over the first 20.0 seconds?

Hint C.1 **Definition and graphical interpretation of average acceleration**

The average acceleration a_{av} of a particle that travels along a straight line in a time interval Δt is the ratio of the change in velocity Δv experienced by the particle to the time interval Δt, or

$$a_{av} = \frac{\Delta v}{\Delta t}.$$

In a v vs. t graph, then, the average acceleration equals the slope of the line connecting the two points representing the initial and final velocities.

Hint C.2 **Slope of a line**

Hint not displayed

Express your answer in meters per second per second.

ANSWER: a_{av} = **0.075** m/s^2
Correct

The average acceleration of a particle between two instants of time is the slope of the line connecting the two corresponding points in a v vs. t graph.

Part D

What is the instantaneous acceleration a of the particle at $t = 45.0$ s?

Hint D.1 **Graphical interpretation of instantaneous acceleration**

Hint not displayed

Hint D.2 **Slope of a line**

Hint not displayed

ANSWER:

a =
- 1 m/s^2
- 0.20 m/s^2
- -0.20 m/s^2
- 0.022 m/s^2
- -0.022 m/s^2

Correct

The instantaneous acceleration of a particle at any point on a v vs. t graph is the slope of the line tangent to the curve at that point. Since in the last 10 seconds of motion, between $t = 40.0$ s and $t = 50.0$ s, the curve is a straight line, the tangent line is the curve itself. Physically, this means that the instantaneous acceleration of the particle is *constant* over that time interval. This is true for any motion where velocity increases linearly with time. In the case at hand, can you think of another time interval in which the acceleration of the particle is constant?

Now that you have reviewed how to plot variables as a function of time, you can use the same technique and draw an acceleration vs. time graph, that is, the graph of (instantaneous) acceleration as a function of time. As usual in these types of graphs, time t is plotted on the horizontal axis, while the vertical axis is used to indicate acceleration a.

Part E

Which of the graphs shown below is the correct acceleration vs. time plot for the motion described in the previous parts?

Hint E.1 **How to approach the problem**

Hint not displayed

Hint E.2 **Find the acceleration in the first 20 s**

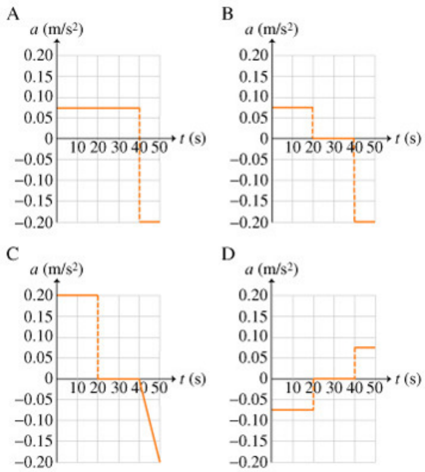
Hint not displayed

Hint E.3 **Find the acceleration in the second 20 s**

Hint not displayed

Hint E.4 **Find the acceleration in the last 10 s**

Hint not displayed



ANSWER:
- Graph A
- Graph B
- Graph C
- Graph D

Correct

In conclusion, graphs of velocity as a function of time are a useful representation of straight-line motion. If read correctly, they can provide you with all the information you need to study the motion.

Tossing Balls off a Cliff

Learning Goal: To clarify the distinction between speed and velocity, and to review qualitatively one-dimensional kinematics.

A woman stands at the edge of a cliff, holding one ball in each hand. At time t_0, she throws one ball straight up with speed v_0 and the other straight down, also with speed v_0.

For the following questions neglect air resistance. Pay particular attention to whether the answer involves "absolute" quantities that have only magnitude (e.g., speed) or quantities that can have either sign (e.g., velocity). Take upward to be the positive direction.

Part A

If the ball that is thrown downward has an acceleration of magnitude a at the instant of its release (i.e., when there is no longer any force on the ball due to the woman's hand), what is the relationship between a and g, the magnitude of the acceleration of gravity?

ANSWER:

- $a > g$
- $a = g$
- $a < g$

Correct

Part B

Which ball has the greater acceleration at the instant of release?

ANSWER:

- the ball thrown upward
- the ball thrown downward
- Neither; the accelerations of both balls are the same.

Correct

Part C

Which ball has the greater speed at the instant of release?

Hint C.1 **Consider the initial speeds**

Hint not displayed

ANSWER:

- the ball thrown upward
- the ball thrown downward
- Neither; the speeds are the same.

Correct

Part D

Which ball has the greater average speed during the 1-s interval after release (assuming neither hits the ground during that time)?

Hint D.1 **How to approach the problem**

Hint not displayed

ANSWER:

- the ball thrown upward
- the ball thrown downward
- Neither; the average speeds of both balls are the same.

Correct

Part E

Which ball hits the ground with greater speed?

ANSWER:

- the ball thrown upward
- the ball thrown downward
- Neither; the balls hit the ground with the same speed.

Correct

Problem 2.13

A speed skater moving across frictionless ice at 8.00 m/s hits a 5.00-m-wide patch of rough ice. She slows steadily, then continues on at 5.90 m/s.

Part A

What is her acceleration on the rough ice?

ANSWER: -2.92 m/s^2

Correct

Problem 2.19

A skier is gliding along at $3.0\ m/s$ on horizontal, frictionless snow. He suddenly starts down a $10°$ incline. His speed at the bottom is $15\ m/s$.

Part A

What is the length of the incline?

ANSWER: **63.5** m *Correct*

Part B

How long does it take him to reach the bottom?

ANSWER: **7.05** s *Correct*

A Flea in Flight

In this problem, you will apply kinematic equations to a jumping flea. Take the magnitude of free-fall acceleration to be 9.80 $\mathrm{m/s^2}$. Ignore air resistance.

Part A

A flea jumps straight up to a maximum height of 0.430 m. What is its initial velocity v_0 as it leaves the ground?

Hint A.1 **Finding the knowns and unknowns**

Hint not displayed

Hint A.2 **Determine which kinematic equation to use**

Decide which kinematic equation makes the solution of this problem easiest. That is, look for an equation that contains the variable you are solving for and in which all the other variables are known.

ANSWER:

- ○ $v_1 = v_0 + a_y t$
- ○ $y_1 = y_0 + v_0 t + \frac{1}{2} a_y t^2$
- ◉ $v_1^2 = v_0^2 + 2a_y(y_1 - y_0)$
- ○ $y_1 - y_0 = \left(\frac{v_0 + v_1}{2}\right) t$

Correct

Now substitute the known quantities into this equation and find v_0, the variable that you are looking for.

Hint A.3 **Some algebra help**

Hint not displayed

Express your answer in meters per second to three significant figures.

ANSWER: v_0 = **2.90** $\mathrm{m/s}$ *Correct*

Part B

How long is the flea in the air from the time it jumps to the time it hits the ground?

Hint B.1 **How to approach the problem**

Hint not displayed

Hint B.2 **Find the time from the ground to the flea's maximum height**

Hint not displayed

Hint B.3 **Find the time from the flea's maximum height to the ground**

Hint not displayed

Express your answer in seconds to three significant figures.

ANSWER: time in air = **0.592** s *Correct*

Notice that the time for the flea to rise to its maximum height is equal to the time it takes for it to fall from that height back to the ground. This is a general feature of projectile motion (any motion with $a = -g$) when air resistance is neglected and the landing point is at the same height as the launch point.

There is also a way to find the total time in the air in one step: just use

$$y = y_0 + v_0 t + \frac{1}{2} a_y t^2$$

and realize that you are looking for the value of t for which $y = y_0$.

Problem 2.25

A particle's position on the *x*-axis is given by the function $x = (t^2 - 5.00\,t + 6.00)\ \mathrm{m}$, where t is in s.

Part A

Where is the particle when v_x = 6.00 $\mathrm{m/s}$?

ANSWER: **8.75** *Correct* m

Rocket Height

A rocket, initially at rest on the ground, accelerates straight upward from rest with constant acceleration 58.8 $\mathrm{m/s^2}$. The acceleration period lasts for time 10.0 s until the fuel is exhausted. After that, the rocket is in free fall.

Part A

Find the maximum height $y_{\rm max}$ reached by the rocket. Ignore air resistance and assume a constant acceleration due to gravity equal to 9.8 $\mathrm{m/s^2}$.

Hint A.1 **How to approach the problem**

Hint not displayed

Hint A.2 **Find the height reached during the fueled part of the motion**

Find the height $y_{\rm fuel}$ above the ground at which the rocket exhausts its fuel.

Hint A.2.1 **Knowns and unknowns**

Hint not displayed

Hint A.2.2 **Determine which kinematic equation to use**

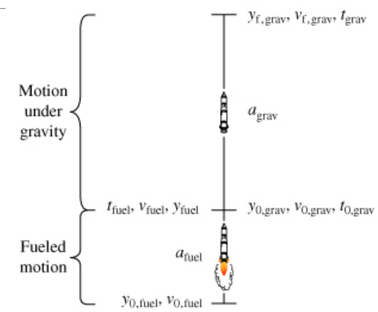
Hint not displayed

Answer numerically in units of meters.

ANSWER: $y_{\rm fuel}$ = **2940** *Correct* m

Note that the upward acceleration of the rocket results from both the thrust of the engine and from the force due to gravity; thus, the existence of gravity is already "taken into account" in the statement of the problem.

You can now either find the total height that the rocket reaches or first determine the additional vertical distance the rocket travels after it runs out of fuel and add this value to the value you found for $y_{\rm fuel}$. Since you don't know the time it takes for the rocket to reach its maximum height, you must determine the quantities that you do know for this part of the motion: the initial velocity $v_{\rm 0,grav}$, the final velocity $v_{\rm f,grav}$, and the acceleration $a_{\rm grav}$. Look at the figure for a clearer picture.



Hint A.3 **Find the initial velocity, the final velocity, and the acceleration for the "free-fall" part of the motion**

What are $v_{\rm 0,grav}$, $v_{\rm f,grav}$, and $a_{\rm grav}$ for the second part of the motion?

Hint A.3.1 **What is the initial velocity?**

Hint not displayed

Hint A.3.2 **What is the acceleration?**

Hint not displayed

Hint A.3.3 **What is the final velocity?**

Hint not displayed

Write your answer numerically in the order $v_{\rm 0,grav}$, $v_{\rm f,grav}$, $a_{\rm grav}$, separated by commas as shown, in SI units.

ANSWER: $v_{\rm 0,grav}$, $v_{\rm f,grav}$, $a_{\rm grav}$ = ***Answer not displayed*** SI units

Hint A.4 **Determine which kinematic equation to use**

Hint not displayed

Write your answer numerically in units of meters.

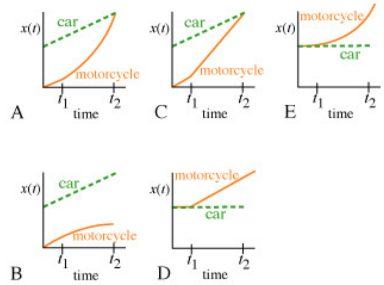
ANSWER: $y_{\rm max}$ = **2.06×10^4** *Correct* m

A Motorcycle Catches a Car

A motorcycle is following a car that is traveling at constant speed on a straight highway. Initially, the car and the motorcycle are both traveling at the same speed of 45.0 mph, and the distance between them is 56.0 m. After t_1 = 4.00 s, the motorcycle starts to accelerate at a rate of 8.00 $\mathrm{m/s^2}$. The motorcycle catches up with the car at some time t_2.

Part A

Which of the graphs correctly displays the positions of the motorcycle and car as functions of time?



Hint A.1 **Describe the graph of the motorcycle's position**

Hint not displayed

Hint A.2 **The relative positions of the two vehicles**

Hint not displayed

ANSWER:

- (•) a
- ( ) b
- ( ) c
- ( ) d
- ( ) e

Correct

Part B

How long does it take from the moment when the motorcycle starts to accelerate until it catches up with the car? In other words, find $t_2 - t_1$.

Hint B.1 **Using a moving reference frame**

For this part, the important quantity is the *relative position*, or the *separation* of the two vehicles. You can consider the motion in a frame of reference that moves with the constant speed of the car (45.0 $\rm mph$). In this frame of reference, the car is standing still, both vehicles have zero initial speed, and so the calculations are simpler. With the car at zero speed and the initial speed of the motorcycle zero, the problem reduces to finding how long it takes the motorcycle to cover a distance of 56.0 $\rm m$ starting at zero velocity with an acceleration of 8.00 $\rm m/s^2$.

However, if you don't feel comfortable with this approach, the rest of the hints for this part will help you with a more traditional method based on the positions of car and motorcycle with respect to the ground as functions of time.

Hint B.2 **Find the initial conditions for the position of the car**

If the initial conditions are known at time t_1, and the motion is one of constant acceleration, the equation for the position of the car at time t_2 is

$$x_{\rm c}(t_2) = x_{1,{\rm c}} + v_{1,{\rm c}}(t_2 - t_1) + \frac{1}{2}a_{\rm c}(t_2 - t_1)^2,$$

where $x_{\rm c}(t)$ is the positon of the car as a function of time, $x_{1,{\rm c}}$ is its position at time t_1, $v_{1,{\rm c}}$is the car's velocity at time t_1, and $a_{\rm c}$ is the car's constant acceleration. (If $t_1 = 0$, the equations become more familiar.) Let us choose a frame of reference in which at time t_1, the motorcycle is at position $x_{1,{\rm m}} = 0$. What are the values of $x_{1,{\rm c}}$, $v_{1,{\rm c}}$, and $a_{\rm c}$ that you should use in the above equation?

Enter your answer in the order $x_{1,{\rm c}}, v_{1,c}, a_{\rm c}$, separated by commas as shown, in units of $\rm meters$, $\rm mph$, and $\rm m/s^2$, respectively.

ANSWER: $x_{1,{\rm c}}, v_{1,c}, a_{\rm c}$ = ***Answer not displayed*** $\rm m, mph, m/s^2$

Hint B.3 **Find the initial conditions for the position of the motorcycle**

If initial conditions are known at time t_1, and the motion is one of constant acceleration, the equation for the position of the motorcycle at time t_2 is

$$x_{\rm m}(t_2) = v_{1,{\rm m}}(t_2 - t_1) + \frac{1}{2}a_{\rm m}(t_2 - t_1)^2,$$

where the meaning of the symbols is analogous to that of Part B.2. Observe that there is no term involving the initial position, because here we have assumed that at time t_1, the motorcycle is at position $x_{1,{\rm m}} = 0$. What are the values of $v_{1,{\rm m}}$ and $a_{\rm m}$ that you should use in the above equation?

Enter your answer in the order $v_{1,m}, a_{\rm m}$, separated by commas as shown, in units of $\rm mph$ and $\rm m/s^2$ respectively.

ANSWER: $v_{1,m}, a_{\rm m}$ = ***Answer not displayed*** $\rm mph, m/s^2$

Hint B.4 **Solving for the time**

At time t_2, the car and motorcycle must be at the same position, since they are side by side. This means that you can set $x_{\rm c}(t_2)$ and $x_{\rm m}(t_2)$, the positions of the car and motorcycle at time t_2, equal to each other, and then solve for the quantity $t_2 - t_1$. You should find that some terms cancel out on either side of the equation, which will make your calculations simpler.

Express the time numerically in seconds.

ANSWER: $t_2 - t_1$ = **3.74** s

Correct

Part C

How far does the motorcycle travel from the moment it starts to accelerate (at time t_1) until it catches up with the car (at time t_2)?

Hint C.1 **Find the initial conditions for the position of the motorcycle**

If the initial conditions are known at time t_1, and the motion has constant acceleration, the equation for the position of the motorcycle at time t_2 is

$$x_{\rm m}(t_2) = v_{1,\rm m}(t_2 - t_1) + \frac{1}{2}a_{\rm m}(t_2 - t_1)^2,$$

as discussed in Part B.3. Here we have again assumed that at time t_1, the motorcycle is at position $x_{1,\rm m} = 0$. What are the values of $v_{1,\rm m}$ and $a_{\rm m}$ that you should use in the above equation?

Enter your answer in the order $v_{1,\rm m}, a_{\rm m}$, separated by commas as shown, in units of $\rm m/s$ and $\rm m/s^2$ respectively.

ANSWER: $v_{1,\rm m}, a_{\rm m}$ = **20.0,8.00** $\rm m/s$ and $\rm m/s^2$
Correct

From Part B, you should know $(t_2 - t_1)$, which you can substitute into the above equation in order to calculate $x_{\rm m}(t_2)$.

Answer numerically in meters.

ANSWER: $x_{\rm m}(t_2)$ = **131** m
Correct

Score Summary:

Your score on this assignment is 118.7%.
You received 49.94 out of a possible total of 50 points, plus 9.4 points of extra credit.