Chapter 18 Homework
Due: 9:00am on Wednesday, December 9, 2009

Note: To understand how points are awarded, read your instructor's Grading Policy.

Velocity and Energy Scaling

Hydrogen molecules have a mass of $2u$ and oxygen molecules have a mass of $32u$, where $u$ is defined as an atomic mass unit ($u = 1.660540 \times 10^{-27}$ kg). Compare a gas of hydrogen molecules to a gas of oxygen molecules.

Part A
At what gas temperature $T_h$ would the average translational kinetic energy of a hydrogen molecule be equal to that of an oxygen molecule in a gas of temperature 300 K?

Hint A.1 Find the energy associated with one degree of freedom
The Equipartition Theorem states that the average energy associated with each degree of freedom is $\frac{1}{2}k_BT$. Therefore, what is the average contribution of motion in the $x$ direction to the kinetic energy?

Hint A.1.1 Equipartition Theorem
For the translational kinetic energy of gas particles, the Equipartition Theorem states

$$\frac{1}{2}m(v_x^2) = \frac{1}{2}m(v_y^2) = \frac{1}{2}m(v_z^2) = \frac{1}{2}k_BT,$$

where $m$ is the mass of each gas particle.

Express the average energy due to the $x$ motion in terms of $k_B$ and $T$.

ANSWER: $\frac{1}{2}m(v_x^2) = \frac{3}{2}k_BT$ Correct

Hint A.2 Total translational kinetic energy
Add the contributions from the $x$, $y$, and $z$ degrees of freedom to find the total translational kinetic energy $E_{k_total}$.

Hint A.2.1 Total translational energy in terms of velocities
We can write the total translational energy as follows:

$$E_{k_total} = \frac{1}{2}m(v_x^2) + \frac{1}{2}m(v_y^2) + \frac{1}{2}m(v_z^2).$$

Express the total translational kinetic energy in terms of $k_B$, $T$, and $m$.

ANSWER: $E_{k_total} = \frac{3}{2}k_BT$ Correct

Good! Now notice that the expression involves only the temperature $T$, and not the mass of the molecules, $m$.

Express the temperature numerically in kelvins.

ANSWER: $T_h = 300$ Correct K

Part B
At what gas temperature $T_{rms}$ would the root-mean-square (rms) speed of a hydrogen molecule be equal to that of an oxygen molecule in a gas at 300 K?

Hint B.1 Find the rms speed
From the Equipartition Theorem, you can determine that

$$\frac{1}{2}m(v^2) = 3\left(\frac{3}{2}k_BT\right),$$

where the factor of 3 results from the fact that the molecule can move in three possible directions. From this equation, find $v_{rms} = \sqrt{(v^2)}$.

Express your answer in terms of $k_B$, $T$, and $m$.

ANSWER: $v_{rms} = \sqrt{(v^2)} = \sqrt{3k_BT/m}$ Correct

State your answer numerically, in kelvins, to the nearest integer.

ANSWER: $T_{rms} = 19$ Correct K

Note that 19 K is is a very low temperature. At this temperature, molecular hydrogen still behaves approximately like an ideal gas, as do helium and neon, but all other gases would be frozen solid!
Integrated circuits are manufactured in vacuum chambers in which the air pressure is \(1.40 \times 10^{-10}\) mm Hg. What are (a) the number density and (b) the mean free path of a molecule? Assume \(T = 20.0\) °C.

Part A

**ANSWER:**

\[
\begin{align*}
\text{4.62} \times 10^7 & \text{ m}^{-3} \\
\text{Correct}
\end{align*}
\]

Part B

**ANSWER:**

\[
\begin{align*}
\text{1.22} \times 10^8 & \text{ m} \\
\text{Correct}
\end{align*}
\]

---

**Dust Equipartitions**

Small dust particles suspended in air seem to dance randomly about, a phenomenon called *Brownian motion.* For this problem you will need to know Boltzmann's constant: \(k_B = 1.38 \times 10^{-23}\) J/K.

Part A

What would you expect the mean translational kinetic energy \(K\) of such particles to be if they are in air at a temperature of 290 K?

**Hint A.1** Kinetic energy in terms of average velocity

**Hint A.2** Equipartition Theorem

**Hint A.3** Use the Equipartition Theorem

Express the mean translational kinetic energy numerically, in joules, to two significant figures. Note that \(10^{-23}\) has been factored out already to make your answer simpler.

**ANSWER:**

\[
\begin{align*}
K &= 6.0 \times 10^{-21} \text{ J} \\
\text{Correct}
\end{align*}
\]

Part B

Find an expression for the rms (root-mean-square) speed \(v_{\text{rms}}\) of these particles, assuming them to be spheres of diameter \(d\) and density \(\rho\).

**Hint B.1** Find the rms speed in terms of temperature

**Hint B.2** Find the mass of the particles

Express the rms speed in terms of \(k_B\), \(T\), \(d\), \(\rho\), and \(\pi\).

**ANSWER:**

\[
\begin{align*}
v_{\text{rms}} &= \sqrt{\frac{3k_BT}{4\pi(\frac{d}{2})^3 \rho}} \\
\text{Correct}
\end{align*}
\]

Part C

Now calculate the rms (root-mean-square) speed \(v_{\text{rms}}\) of these particles, assuming them to be spheres of diameter \(\phi = 1\) μm and density \(2 \text{ g/cm}^3 = 2000 \text{ kg/m}^3\). The mass of such a dust particle is \(1.31 \times 10^{-18}\) kg.

Express your answer in millimeters per second to one decimal place only.

**ANSWER:**

\[
\begin{align*}
v_{\text{rms}} &= 0.3 \text{ mm/s} \\
\text{Correct}
\end{align*}
\]

This speed is several orders of magnitude smaller than the typical velocities of gas molecules at this temperature (which are of the order of hundreds of meters per second). This is simply because the mass of these particles is much larger than the mass of typical gas molecules. For particles larger than the ones described here, the weight can no longer be ignored. Such particles tend to settle quite quickly on account of their weight. Then such a calculation is no longer valid.

---

**Problem 18.29**

A 6.30 m x 8.00 m x 2.60 m room contains air at 20°C.

Part A

What is the room's thermal energy?

**ANSWER:**

\[
\begin{align*}
3.32 \times 10^7 & \text{ J} \\
\text{Correct}
\end{align*}
\]

---

**Score Summary:**

Your score on this assignment is 99.3%.

You received 44.7 out of a possible total of 45 points.