Chapter 14 Homework

Due: 9:00am on Thursday, November 19, 2009

Note: To understand how points are awarded, read your instructor's Grading Policy.

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Good Vibes: Introduction to Oscillations

Learning Goal: To learn the basic terminology and relationships among the main characteristics of simple harmonic motion.

Motion that repeats itself over and over is called *periodic motion*. There are many examples of periodic motion: the earth revolving around the sun, an elastic ball bouncing up and down, or a block attached to a spring oscillating back and forth.

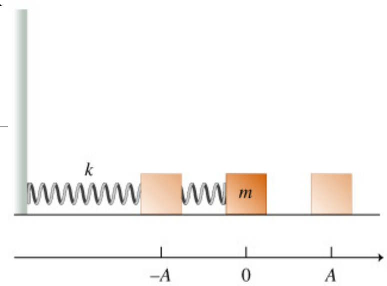
The last example differs from the first two, in that it represents a special kind of periodic motion called *simple harmonic motion*. The conditions that lead to simple harmonic motion are as follows:

- There must be a position of *stable equilibrium*.
- There must be a restoring force acting on the oscillating object. The direction of this force must always point toward the equilibrium, and its magnitude must be directly proportional to the magnitude of the object's displacement from its equilibrium position. Mathematically, the restoring force $\vec{F}$ is given by $\vec{F} = -k\vec{x}$, where $\vec{x}$ is the displacement from equilibrium and k is a constant that depends on the properties of the oscillating system.
- The resistive forces in the system must be reasonably small.

In this problem, we will introduce some of the basic quantities that describe oscillations and the relationships among them.

Consider a block of mass m attached to a spring with force constant k, as shown in the figure. The spring can be either stretched or compressed. The block slides on a frictionless horizontal surface, as shown. When the spring is relaxed, the block is located at $x = 0$. If the block is pulled to the right a distance A and then released, A will be the *amplitude* of the resulting oscillations.

Assume that the mechanical energy of the block-spring system remains unchanged in the subsequent motion of the block.



Part A

After the block is released from $x = A$, it will

ANSWER:

- remain at rest.
- move to the left until it reaches equilibrium and stop there.
- move to the left until it reaches $x = -A$ and stop there.
- move to the left until it reaches $x = -A$ and then begin to move to the right.

Correct

As the block begins its motion to the left, it accelerates. Although the restoring force decreases as the block approaches equilibrium, it still pulls the block to the left, so by the time the equilibrium position is reached, the block has gained some speed. It will, therefore, pass the equilibrium position and keep moving, compressing the spring. The spring will now be pushing the block to the right, and the block will slow down, temporarily coming to rest at $x = -A$.

After $x = -A$ is reached, the block will begin its motion to the right, pushed by the spring. The block will pass the equilibrium position and continue until it reaches $x = A$, completing *one cycle* of motion. The motion will then repeat; if, as we've assumed, there is no friction, the motion will repeat indefinitely.

The time it takes the block to complete one cycle is called the *period*. Usually, the period is denoted T and is measured in seconds.

The *frequency*, denoted f, is the number of cycles that are completed per unit of time: $f = 1/T$. In SI units, f is measured in inverse seconds, or hertz (Hz).

Part B

If the period is doubled, the frequency is

ANSWER:

- unchanged.
- doubled.
- halved.

Correct

Part C

An oscillating object takes 0.10 s to complete one cycle; that is, its period is 0.10 s. What is its frequency f?

Express your answer in hertz.

ANSWER:

f = **10** Hz
Correct

Part D

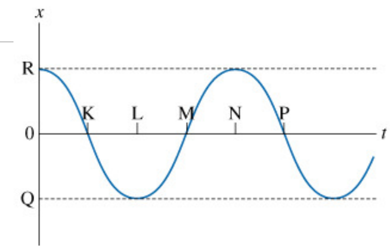
If the frequency is 40 Hz, what is the period T ?

Express your answer in seconds.

ANSWER: T = **0.025** ***Correct*** s

The following questions refer to the figure that graphically depicts the oscillations of the block on the spring.

Note that the vertical axis represents the *x* coordinate of the oscillating object, and the horizontal axis represents time.



Part E

Which points on the *x* axis are located a distance A from the equilibrium position?

ANSWER:

- R only
- Q only
- both R and Q

Correct

Part F

Suppose that the period is T. Which of the following points on the *t* axis are separated by the time interval T?

ANSWER:

- K and L
- K and M
- K and P
- L and N
- M and P

Correct

Now assume that the *x* coordinate of point R is 0.12 m and the *t* coordinate of point K is 0.0050 s.

Part G

What is the period T ?

Hint G.1 **How to approach the problem**

In moving from the point $t = 0$ to the point K, what fraction of a full wavelength is covered? Call that fraction a. Then you can set $aT = 0.005\ \mathrm{s}$. Dividing by the fraction a will give the period T.

Express your answer in seconds.

ANSWER: T = **0.02** ***Correct*** s

Part H

How much time t does the block take to travel from the point of maximum displacement to the opposite point of maximum displacement?

Express your answer in seconds.

ANSWER: t = **0.01** ***Correct*** s

Part I

What distance d does the object cover during one period of oscillation?

Express your answer in meters.

ANSWER: d = **0.48** ***Correct*** m

Part J

What distance d does the object cover between the moments labeled K and N on the graph?

Express your answer in meters.

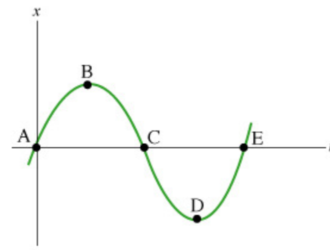
ANSWER: d = **0.36** ***Correct*** m

Position, Velocity, and Acceleration of an Oscillator

Learning Goal: To learn to find kinematic variables from a graph of position vs. time.

The graph of the position of an oscillating object as a function of time is shown.

Some of the questions ask you to determine ranges on the graph over which a statement is true. When answering these questions, choose the *most complete* answer. For example, if the answer "B to D" were correct, then "B to C" would technically also be correct--but you will only recieve credit for choosing the most complete answer.



Part A

Where on the graph is $x > 0$?

ANSWER:

- A to B
- A to C
- C to D
- C to E
- B to D
- A to B and D to E

Correct

Part B

Where on the graph is $x < 0$?

ANSWER:

- A to B
- A to C
- C to D
- C to E
- B to D
- A to B and D to E

Correct

Part C

Where on the graph is $x = 0$?

ANSWER:

- A only
- C only
- E only
- A and C
- A and C and E
- B and D

Correct

Part D

Where on the graph is the velocity $v > 0$?

Hint D.1 **Finding instantaneous velocity**

Instantaneous velocity is the derivative of the position function with respect to time,

$$v(t) = \frac{dx(t)}{dt}.$$

Thus, you can find the velocity at any time by calculating the slope of the x vs. t graph. When is the slope greater than 0 on this graph?

ANSWER:

- A to B
- A to C
- C to D
- C to E
- B to D
- A to B and D to E

Correct

Part E

Where on the graph is the velocity $v < 0$?

ANSWER:

- A to B
- A to C
- C to D
- C to E
- B to D
- A to B and D to E

Correct

Part F

Where on the graph is the velocity $v = 0$?

Hint F.1 **How to tell if $v = 0$**

Hint not displayed

ANSWER:

- A only
- B only
- C only
- D only
- E only
- A and C
- A and C and E
- B and D

Correct

Part G

Where on the graph is the acceleration $a > 0$?

Hint G.1 **Finding acceleration**

Acceleration is the second derivative of the position function with respect to time:

$$a = \frac{d^2x(t)}{dt^2}.$$

This means that the sign of the acceleration is the same as the sign of the curvature of the *x* vs. *t* graph. The acceleration of a curve is negative for downward curvature and positive for upward curvature. Where is the curvature greater than 0?

ANSWER:

- A to B
- A to C
- C to D
- C to E
- B to D
- A to B and D to E

Correct

Part H

Where on the graph is the acceleration $a < 0$?

ANSWER:

- A to B
- A to C
- C to D
- C to E
- B to D
- A to B and D to E

Correct

Part I

Where on the graph is the acceleration $a = 0$?

Hint I.1 **How to tell if $a = 0$**

Hint not displayed

ANSWER:

- A only
- B only
- C only
- D only
- E only
- A and C
- A and C and E
- B and D

Correct

Energy of Harmonic Oscillators

Learning Goal: To learn to apply the law of conservation of energy to the analysis of harmonic oscillators.

Systems in simple harmonic motion, or *harmonic oscillators*, obey the law of conservation of energy just like all other systems do. Using energy considerations, one can analyze many aspects of motion of the oscillator. Such an analysis can be simplified if one assumes that mechanical energy is not dissipated. In other words,

$$E = K + U = \text{constant},$$

where E is the total mechanical energy of the system, K is the kinetic energy, and U is the potential energy.

As you know, a common example of a harmonic oscillator is a mass attached to a spring. In this problem, we will consider a *horizontally* moving block attached to a spring. Note that, since the gravitational potential energy is not changing in this case, it can be excluded from the calculations.

For such a system, the potential energy is stored in the spring and is given by

$$U = \frac{1}{2}kx^2,$$

where k is the force constant of the spring and x is the distance from the equilibrium position.

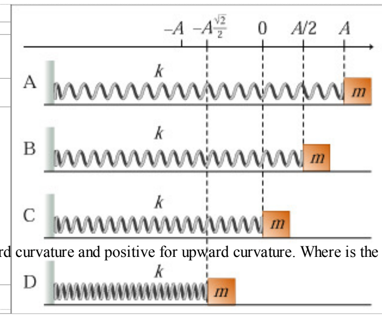
The kinetic energy of the system is, as always,

$$K = \frac{1}{2}mv^2,$$

where m is the mass of the block and v is the speed of the block.

We will also assume that there are no resistive forces; that is, $E = \text{constant}$.

Consider a harmonic oscillator at four different moments, labeled A, B, C, and D, as shown in the figure . Assume that the force constant k, the mass of the block, m, and the amplitude of vibrations, A, are given. Answer the following questions.



Part A

Which moment corresponds to the maximum potential energy of the system?

Hint A.1 **Consider the position of the block**

Hint not displayed

ANSWER:

- A
- B
- C
- D

Correct

Part B

Which moment corresponds to the minimum kinetic energy of the system?

Hint B.1 **How does the velocity change?**

Hint not displayed

ANSWER:

- A
- B
- C
- D

Correct

When the block is displaced a distance A from equilibrium, the spring is stretched (or compressed) the most, and the block is momentarily at rest. Therefore, the maximum potential energy is $U_{\max} = \frac{1}{2}kA^2$. At that moment, of course, $K = K_{\min} = 0$. Recall that $E = K + U$. Therefore,

$$E = \frac{1}{2}kA^2.$$

In general, the mechanical energy of a harmonic oscillator equals its potential energy at the maximum or minimum displacement.

Part C

Consider the block in the process of oscillating.

ANSWER:

If the kinetic energy of the block is increasing, the block *must* be

- ○ at the equilibrium position.
- ○ at the amplitude displacement.
- ○ moving to the right.
- ○ moving to the left.
- ○ moving away from equilibrium.
- ◉ moving toward equilibrium.

Correct

Part D

Which moment corresponds to the maximum kinetic energy of the system?

Hint D.1 **Consider the velocity of the block**

Hint not displayed

ANSWER:

- ○ A
- ○ B
- ◉ C
- ○ D

Correct

Part E

Which moment corresponds to the minimum potential energy of the system?

Hint E.1 **Consider the distance from equilibrium**

Hint not displayed

ANSWER:

- ○ A
- ○ B
- ◉ C
- ○ D

Correct

When the block is at the equilibrium position, the spring is not stretched (or compressed) at all. At that moment, of course, $U = U_{\rm min} = 0$. Meanwhile, the block is at its maximum speed ($v_{\rm max}$). The maximum kinetic energy can then be written as $K_{\rm max} = \frac{1}{2}mv_{\rm max}^2$. Recall that $E = K + U$ and that $U = 0$ at the equilibrium position. Therefore,

$$E = \frac{1}{2}mv_{\rm max}^2.$$

Recalling what we found out before,

$$E = \frac{1}{2}kA^2,$$

we can now conclude that

$$\frac{1}{2}kA^2 = \frac{1}{2}mv_{\rm max}^2,$$

or

$$v_{\rm max} = \sqrt{\frac{k}{m}}A = \omega A.$$

Part F

At which moment is $K = U$?

Hint F.1 **Consider the potential energy**

At this moment, $U = \frac{1}{2}U_{\rm max}$. Use the formula for $U_{\rm max}$ to obtain the corresponding distance from equilibrium.

ANSWER:

- ○ A
- ○ B
- ○ C
- ◉ D

Correct

Part G

Find the kinetic energy K of the block at the moment labeled B.

Hint G.1 **How to approach the problem**

Find the potential energy first; then use conservation of energy.

Hint G.2 **Find the potential energy**

Find the potential energy U of the block at the moment labeled B.

Express your answer in terms of k and A.

ANSWER:

$U = \frac{1}{8}kA^2$

Correct

Using the facts that the total energy $E = \frac{1}{2}kA^2$ and that $E = K + U$, you can now solve for the kinetic energy K at moment B.

Express your answer in terms of k and A.

ANSWER:

$K = \frac{3}{8}kA^2$

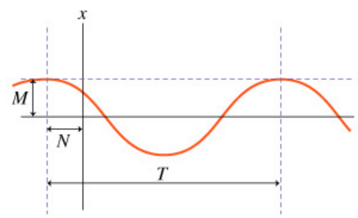
Correct

Cosine Wave

The graph shows the position x of an oscillating object as a function of time t. The equation of the graph is

$$x(t) = A\cos(\omega t + \phi),$$

where A is the amplitude, ω is the angular frequency, and ϕ is a phase constant. The quantities M, N, and T are measurements to be used in your answers.



Part A

What is A in the equation?

Hint A.1 **Maximum of $x(t)$**

Hint not displayed

ANSWER:

- T
- M
- $2M$
- M/T
- $T/2$

Correct

Part B

What is ω in the equation?

Hint B.1 **Period**

Hint not displayed

ANSWER:

- T
- M
- $2\pi T$
- $2\pi/T$
- $2/T$
- $1/T$

Correct

Part C

What is ϕ in the equation?

Hint C.1 **Using the graph and trigonometry**

What is x equal to when $t = -N$? Use your result for ω to solve for ϕ in terms of T, M, and N.

Hint C.2 **Using the graph and Part B**

You might be able to find ϕ in terms of ω and then use your result from Part B.

ANSWER:

- N
- $T - N$

- ◉ $2\pi N/T$
- ○ $-2\pi N/T$
- ○ $\arccos(2\pi N/T)$

Correct

Analyzing Simple Harmonic Motion

This applet shows two masses on springs, each accompanied by a graph of its position versus time.

Part A

What is an expression for $x_1(t)$, the position of mass I as a function of time? Assume that position is measured in meters and time is measured in seconds.

Hint A.1 **How to approach the problem**

The most general form of a sinusoidal wave is

$$x(t) = \pm A\sin(\omega t + \phi) \text{ or}$$

$$x(t) = \pm A\cos(\omega t + \phi'),$$

depending upon whether you write the equation using the sine or cosine. Notice that the amplitude A and the angular frequency ω will be the same regardless of whether you choose to use sine or cosine. The phase ϕ, however, will be different, since for any α, we have the relation $\cos(\alpha) = -\sin(\alpha - \pi/2)$.

The amplitude A and angular frequency ω can be determined directly from the graph. The decision whether to use sine or cosine is more a matter of convenience. Which of the following statements correctly identify a choice of function and phase that could be used to describe this graph?

Check all that apply.

ANSWER:

- ☑ cosine, $\phi' = 0$
- ☐ cosine, $\phi' = -\pi/2$
- ☑ cosine, $\phi' = \pi$
- ☐ sine, $\phi = 0$
- ☑ sine, $\phi = -\pi/2$
- ☐ sine, $\phi = \pi$

Answer Requested

Of the three correct options, the simplest is cosine with phase shift zero, but any of them will give the correct answer.

Hint A.2 **Find the amplitude**

What is the amplitude A of the motion of the first mass?

Hint A.2.1 **Definition of amplitude**

Hint not displayed

Express your answer in meters to two significant figures.

ANSWER: A = 1 m ***Correct***

Hint A.3 **Find the angular frequency**

What is the angular frequency ω of the motion of the first mass?

Hint A.3.1 **Angular frequency and frequency**

Hint not displayed

Express your answer in radians per second to three significant figures.

ANSWER: ω = 12.6 rad/s ***Correct***

Express your answer as a function of t. Express numerical constants to three significant figures.

ANSWER: $x_1(t) = -\cos(12.6t)$ ***Correct***

Part B

What is $x_2(t)$, the position of mass II as a function of time? Assume that position is measured in meters and time is measured in seconds.

Hint B.1 **How to approach the problem**

Hint not displayed

Hint B.2 **Find the amplitude**

Hint not displayed

Hint B.3 **Find the angular frequency**

Hint not displayed

Express your answer as a function of t. Express numerical constants to three significant figures.

ANSWER: $x_2(t) = \frac{1}{2}\cos(2\pi t)$

Correct

Changing the Period of a Pendulum

A simple pendulum consisting of a bob of mass m attached to a string of length L swings with a period T.

Part A

If the bob's mass is doubled, approximately what will the pendulum's new period be?

Hint A.1 **Period of a simple pendulum**

Hint not displayed

ANSWER:

- $T/2$
- T
- $\sqrt{2}T$
- $2T$

Correct

Part B

If the pendulum is brought on the moon where the gravitational acceleration is about $g/6$, approximately what will its period now be?

Hint B.1 **How to approach the problem**

Hint not displayed

ANSWER:

- $T/6$
- $T/\sqrt{6}$
- $\sqrt{6}T$
- $6T$

Correct

Part C

If the pendulum is taken into the orbiting space station what will happen to the bob?

Hint C.1 **How to approach the problem**

Hint not displayed

ANSWER:

- It will continue to oscillate in a vertical plane with the same period.
- It will no longer oscillate because there is no gravity in space.
- It will no longer oscillate because both the pendulum and the point to which it is attached are in free fall.
- It will oscillate much faster with a period that approaches zero.

Correct

In the space station, where all objects undergo the same acceleration due to the earth's gravity, the tension in the string is zero and the bob does not fall relative to the point to which the string is attached.

Gravity on Another Planet

After landing on an unfamiliar planet, a space explorer constructs a simple pendulum of length 53.0 cm. The explorer finds that the pendulum completes 94.0 full swing cycles in a time of 131 s.

Part A

What is the value of the acceleration of gravity on this planet?

Hint A.1 **How to approach the problem**

Calculate the period of the pendulum, and use this to calculate the gravitational acceleration on the planet.

Hint A.2 **Calculate the period**

Calculate the period T of the pendulum.

Express your answer in seconds.

ANSWER: T = **1.39** s

Correct

Hint A.3 **Equation for the period**

The period of a simple pendulum is given by the equation $T = 2\pi\sqrt{L/g_{\rm planet}}$, where L is the length of the pendulum and $g_{\rm planet}$ is the gravitational acceleration on the planet.

Express your answer in meters per second per second.

ANSWER: g_{planet} = **10.8** m/s^2
Correct

Problem 14.26

A 2.0 g spider is dangling at the end of a silk thread. You can make the spider bounce up and down on the thread by tapping lightly on his feet with a pencil. You soon discover that you can give the spider the largest amplitude on his little bungee cord if you tap exactly once every second.

Part A

What is the spring constant of the silk thread?

ANSWER: **7.90×10^{-2}** N/m
Correct

Damped Egg on a Spring

A 50.0-g hard-boiled egg moves on the end of a spring with force constant $k = 25.0 \text{ N/m}$. It is released with an amplitude 0.300 m. A damping force $F_x = -bv$ acts on the egg. After it oscillates for 5.00 s, the amplitude of the motion has decreased to 0.100 m.

Part A

Calculate the magnitude of the damping coefficient b.

Hint A.1 **How damped is it?**

The system described above is ___________.

Hint A.1.1 **How to determine damping**

Hint not displayed

ANSWER:

- critically damped
- overdamped
- underdamped

Correct

Hint A.2 **What formula to use**

In this problem, the motion is described by the general equation for an underdamped oscillator,

$$x = Ae^{-bt/2m}\cos(\omega' t + \phi),$$

where

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}},$$

x is position, and t is time. The displacement is thus a product of a oscillating cosine term and a damping term $A_1(t)$. This equation is the solution to the damped oscillator equation $m\ddot{x} = -kx - b\dot{x}$.

Hint A.3 **Find the amplitude**

What is $A_1(t)$, the amplitude as a function of time? Use A_0 for the initial displacement of the system and m for the mass of the egg.

Hint A.3.1 **Initial amplitude**

Hint not displayed

Give your answer in terms of A_0, m, b, and t.

ANSWER: $A_1(t) = A_0 e^{-\frac{bt}{2m}}$
Correct

Now evaluate $A_1(5.00\ s)$ and set your answer equal to 0.100 m as given in the problem introduction. Finally, solve for b.

Hint A.4 **Solving for y in e^y**

If $e^y = C$ then $y = \ln C$.

Express the magnitude of the damping coefficient numerically in kilograms per second, to three significant figures.

ANSWER: b = **2.20×10^{-2}** kg/s
Correct

Problem 14.70

An oscillator with a mass of 500 g and a period of 0.900 s has an amplitude that decreases by 2.20% during each complete oscillation.

Part A

If the initial amplitude is 10.6 cm, what will be the amplitude after 37.0 oscillations?

ANSWER: **4.65** *Correct* cm

Part B

At what time will the energy be reduced to 54.0% of its initial value?

ANSWER: **12.5** *Correct* s

Score Summary:

Your score on this assignment is 105%.
You received 63.47 out of a possible total of 65 points, plus 4.8 points of extra credit.